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Charm physics with physical light and strange quarks using domain wall fermions

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We present a study of charm physics using RBC/UKQCD's 2+1 flavour physical point Domain Wall Fermion ensembles for the light quarks as well as for the valence charm quark. After a brief motivation of domain wall fermions as a suitable heavy quark discretisation we will show first results for masses and matrix elements.

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β	a/fm	L/a	T/a	m_π/MeV	$m_\pi L$	β	a/fm	L/a	T/a	m_π/MeV	$m_\pi L$
2.13	0.11	24	64	431	5.8	2.25	0.08	32	64	360	4.8
2.13	0.11	24	64	341	4.6	2.25	0.08	32	64	304	4.1
2.13	0.11	48	96	139	3.8	2.25	0.08	64	128	139	3.9

Table 1: Parameters for ensembles processed so far: β is the gauge coupling, a is the lattice spacing, L and T are the spatial and time-extent of the lattice, respectively, m_π is the pion mass.

1. Introduction

This talk presented our efforts towards a charmed meson physics program on RBC/UKQCD's $N_f = 2+1$ domain wall ensembles using the same discretisation for the light and the charm quarks. Compared to lattice QCD for light quark physics, simulations for charm are comparatively scarce (cf. the FLAG report [1]). One of the reasons is that the charm quark sets a mass scale which until recently was very difficult to reconcile with the energy scale set by the light quarks in a fully relativistic and dynamical lattice setup. In fact, covering both energy scales at the same time still poses a challenge due to the large lattice size that one needs to simulate in order to keep both finite volume and cutoff effects reliably under control.

Our programme consists of first studying domain wall fermions as a charm quark discretisation within the quenched approximation. Let us emphasise that the intention is not to make any phenomenologically relevant predictions from within this framework but rather to understand the behaviour of domain wall fermions for heavy quarks and the continuum limit in detail. To this end we created ensembles and measurements for mesonic matrix elements for cutoffs in the range of 2-6GeV. While a similar study with dynamical domain wall quarks is still out of question for the foreseeable future we expect a number of properties to be very similar. In particular, we expect that the continuum limit scaling observed in the quenched theory over a large range of lattice spacings will be qualitatively the same also beyond the quenched approximation. We can therefore use the scaling behaviour found in the quenched theory to constrain contributions to the scaling from higher orders in the lattice spacing when analysing data in the dynamical case but on a smaller range of lattice cutoffs. Our exploratory quenched studies have been covered in Tsang's and Cho's talks at this conference [2, 3]. The main result is the observation of a large region of heavy quark masses for which the automatic $O(a)$-improvement of domain wall quarks is maintained and spectral quantities and matrix elements extrapolate to the continuum limit linearly in a^2. We found that by tuning the domain wall height, a free parameter in this discretisation, a reduction in the size of cutoff effects can be achieved. The experience gained in the quenched case is now being applied to a charm phenomenology program using dynamical gauge field ensembles. This talk reports on the status of these simulations.

2. Our strategy

Table 1 gives an overview of RBC/UKQCD's domain wall ensembles which we are currently analysing for this charm study. The ensembles to the left have a lattice cutoff of 1.7-1.8GeV (coarse) and the ones to the right 2.4GeV (medium). We are currently generating an additional en-

semble aiming at a third lattice spacing with $a^{-1} \approx 2.8$GeV and with a pion mass around 200MeV (fine). The global strategy can be summarised as follows: we will compute heavy-light, heavy-strange and heavy-heavy meson observables (masses, leptonic and semileptonic decay matrix elements, bag parameters) on all ensembles and also the vector two-point function relevant for computing the anomalous magnetic moment of the muon. Based on the findings in the quenched study we are certain that predictions for these quantities for the physical charm quark mass and even heavier, towards the bottom quark mass, can be made on the medium and fine ensembles with small cutoff effects and functioning automatic $O(a)$-improvement. This may not be the case on the coarse ensemble where the range in quark masses where we expect to have a good control over cutoff effects doesn't allow to simulate directly at charm. On the coarse ensemble the bare input quark mass corresponding to charm is above 0.45 and hence in the part of parameter space where our numerical evidence from the quenched theory suggests the beakdown of $O(a)$-improvement [2]. We therefore consider a strategy where results for all quantities with heavy quark masses slightly smaller than charm are made in the continuum limit. This is then followed by a small extrapolation in the heavy quark mass towards the physical mass of charm. This does not preclude considering a global analysis ansatz taking advantage of the results for heavyier quark masses on the medium and fine ensemble once the latter has been generated and analysed.

In the following we present first results for the heavy-light and heavy-strange decay constant.

3. Choice of parameters

Our gauge field ensembles represent QCD with $N_f = 2+1$ dynamical flavours at two different lattice spacings [4, 5, 6, 7]. The basic parameters of these ensembles are listed in Table 1. For the lattices with physical light quark masses we use the Iwasaki gauge action [8, 9] and the domain wall fermion action [10, 11] with the Möbius-kernel [12, 13, 14]. For the unphysical quark mass ensembles we use the *standard* Shamir-kernel. The difference between both kernels in our implementation [7] is a rescaling such that Möbius domain wall fermions are equivalent to Shamir domain wall fermions at twice the extension in the fifth dimesion. Möbius domain wall fermions are hence cheaper to simulate while providing the same level of lattice chiral symmetry. Results from both formulations of domain wall fermions lie on the same scaling trajectory towards the continuum limit where cutoff-effects are of $O(a^2)$. All lattices entering the following analysis have values of $m_\pi L \gtrsim 4$. We therefore expect finite volume effects to be at the percent level.

We use Möbius Domain Wall fermions also for the discretisation of charm quarks where the choice of simulation parameters in the dynamical study is based on our findings from the quenched case. In particular, the choice of the Domain Wall Height turns out to be crucial in reducing discretisation effects. We also found that discretisation effects become sizeable abruptly as the bare input quark mass is chosen larger than $am_h \approx 0.45$. This is also reflected in an increase in the residual mass. The residual mass is a measure for the amount of chiral symmetry violation of domain wall fermions due to the finite extent of the 5th dimension. The axial Ward identity for domain wall fermion takes the form

$$\langle \partial_\mu \mathscr{A}_\mu(x) P(0) \rangle = 2m \langle P(x) P(0) \rangle + 2 \langle J_{5q}(x) P(0) \rangle \,. \tag{3.1}$$

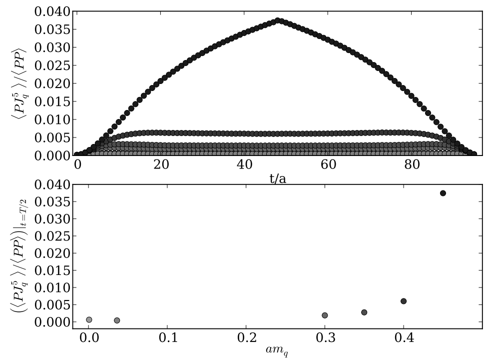

Figure 1: Motivated by the lattice axial Ward Identity the observable in the top panel is constant towards the centre of the lattice for the range of simulation parameters where the domain wall mechanism is functioning well. We found that the residual mass starts increasing severely as the bare input quark mass is chosen beyond 0.45. From this point on it's no longer guaranteed that chiral symmetry is maintained. The bottom plot shows the mass dependence of the residual mass (we took the value of $m_{\rm res}$ at $T/2$).

Here, $\mathscr{A}_\mu$ is the conserved axial current, P is the pseudo scalar density and J_{5q} is also a pseudo-scalar current but it mediates between the boundaries of the 5th dimension and the mid-planes. We define the residual mass as

$$am_{\rm res} = \frac{\sum\limits_x \langle J_{5q}(x)P(0)\rangle}{\sum\limits_x \langle P(x)P(0)\rangle}\,. \tag{3.2}$$

The behaviour of the residual mass as we increase the bare input quark mass is shown in figure 1. For bare quark masses below around $am_h = 0.45$ we see a plateau which is the expected behaviour. For larger input quark masses the behaviour changes drastically indicating the breakdown of the domain wall mechanism. Based on these findings all simulations shown in the following will be for a number of input charm quark masses $am_h \leqq 0.45$. The heaviest charm quark we simulate therefore corresponds to η_c masses of 2.8GeV on the coarse ensemble and 3.3GeV on the medium ensemble (for comparison the physical η_c mass 2.9836(7)GeV [15]).

On the coarser physical point ensemble we generate 48 time-plane complex Z_2 noise sources per configuration and on the finer ensemble 32 time-planes. Meson twopoint functions are then computed using the one-end-trick [16, 17]. On the unphysical point ensemble the number of source planes varies and simulations are still ongoing. One particular concern was the convergence of the conjugate gradient minimiser in the computation of the quark propagator. We employ the time-slice residual [18]

$$r(t) = \frac{|D\psi - \eta|_t}{|\psi|_t} \tag{3.3}$$

where D is the Dirac operator, ψ the solution vector and η the source vector. The norm $|\cdot|_t$ is restricted to the time-slice t. Global residuals will not notice convergence issues at large Euclidean time-distances from the source in a situation where $|\psi|_t$ decays with t over more orders of magnitude than covered by the numerical precision. The time-slice residual is a local (in time) residual that allows to control convergence more reliably.

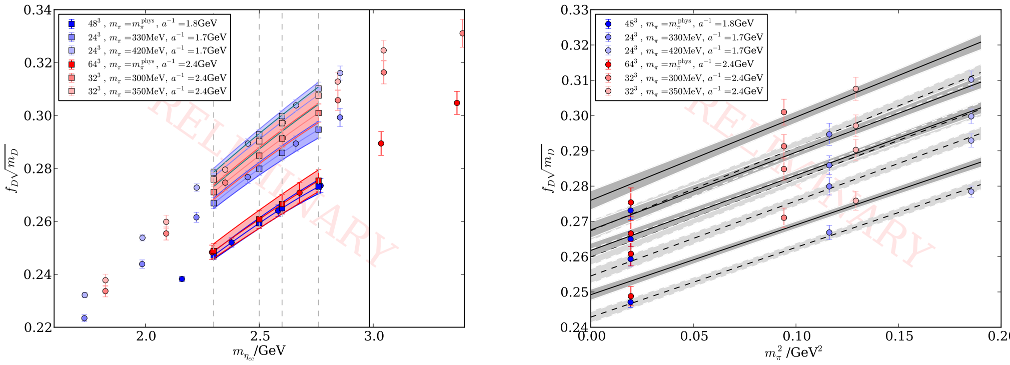

Figure 2: Left: data for the D-meson decay constant vs. the η_c mass. The blue and red bands represent the result of a polynomial fit to the data. The dashed vertical lines indicate the reference η_c-masses which we interpolated to. Right: The results for the decay constant obtained for the reference η_c masses are then interpolated in the pion mass to its physical value. This interpolation is over a very small range given that two of our ensembles have near-physical simulation parameters.

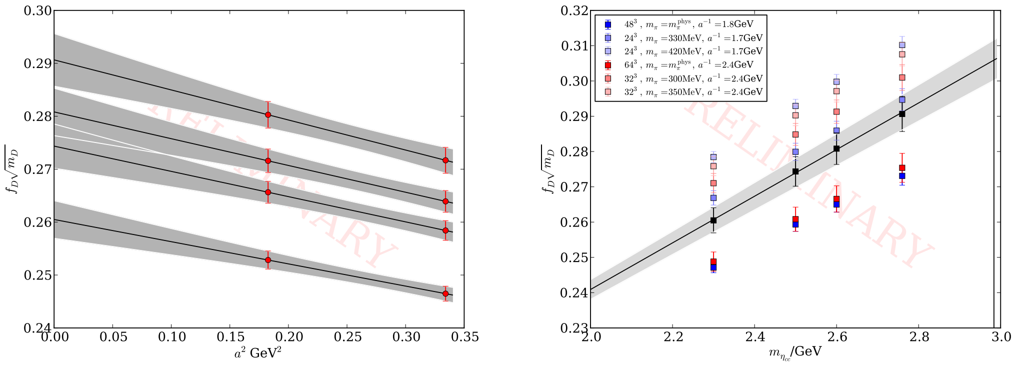

Figure 3: Left: Preliminary result for the continuum extrapolation of the data with physical light quark mass at the reference heavy-quark mass points. Right: Extrapolation of the data in the continuum limit (black squares) to the physical charm point (solid vertical line to the right).

4. Preliminary results

The idea is to compute heavy-light and heavy-strange meson observables for a number of unphysical heavy quark masses below and above the physical charm quark mass as illustrated in the l.h.s plot in figure 2 for the case of the D-meson decay constant. These will then be interpolated to common values for the η_c-mass on each ensemble (the analysis discussed in the following uses the reference masses m_{η_c} =2.3GeV, 2.5GeV, 2.6GeV and 2.76GeV as indicated by the dashed vertical lines). The mass dependence is close to linear and interpolating with polynomials of various orders leads to very similar results. Results from different orders for this interpolation can at a later stage be used to devise an estimate of a systematic error which will likely turn out to be very small. We note that at this stage we do not yet have full statistics and error bars for all ensembles will be much

smaller once the simulations are finalised.

The r.h.s. plot of figure 2 shows the results of the above interpolation plotted against the pion mass. The points to the very left in this plot are from our physical point ensembles. Although these ensembles represent QCD with valence and sea pions close to the physical point we still wish to study interpolation ansätze that allow us to use also the results obtained on the heavier sea quark mass ensembles. Firstly, this will allow for correcting small mistunings in the light quark mass towards the actual physical point. Secondly, in the near term we will not be able to generate fine ensembles with physical sea quarks. Under the reasonable assumption that the light quark mass dependence will suffer only very small cutoff effects our precise knowledge of the mass dependence on the coarse and medium ensembles will allow for extrapolating the result on the fine ensemble towards physical light quark masses. The interpolation shown in the r.h.s. plot in figure 2 are done under the assumption that the slope with m_π^2 is independent of the mass cutoff. With better statistics on the coarse and medium ensemble we will soon be able to make more rigid statements about the validity of this assumption.

The interpolation in the light quark mass is followed by the continuum extrapolation. The l.h.s. plot in figure 3 can only be indicative – a continuum limit with only two points for charm observables is very risky and no quantitative phenomenological conclusions should be drawn from our data at the moment. This will have to wait until we have results for the finer lattice spacing. The r.h.s. plot in figure 3 shows the extrapolation of the results in the continuum limit to the physical charm point (solid vertical line) using a linear ansatz.

5. Conclusions and outlook

In this talk we presented the status of our new charm physics program based on RBC/UKQCD's $N_f = 2+1$ domain wall fermion ensembles with an added valence domain domain wall charm quark. We have shown first results for the D-meson decay constant. As anticipated based on the data from our exploratory quenched study we expect rather mild cutoff effects for our choice of simulation data and judging from the data on our coarse and medium lattice this remains so also in the dynamical case.

We are currently generating an ensemble for a third lattice spacing which will allow for reliably extrapolating our results to the continuum limit. Together with the data for the leptonic D- and D_s decay constant we are also generating heavy-heavy correlators and also the correlation functions relevant for determining the meson bag parameter and semi-leptonic meson decays. Further to charm phenomenology we are studying how to make use of results for heavier than charm quark masses which we are generating in particular on the medium and fine ensemble. To this end we have in mind interpolating with results in the static limit (HQET) but also applying the *ratio method* [19].

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